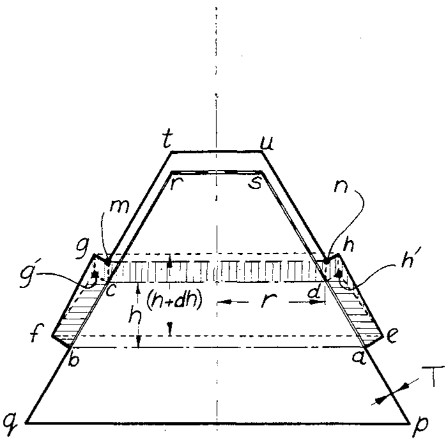

Fig. 1 Conical shell with a transverse element under strain.

The results were based upon height and volume changes, dh and dv, associated with a transverse element of the tank of infinitesimal height h. In the limit $h \rightarrow dL$. However, it has been found that the integrals of these quantities do not, in general, add up to the correct expressions for the total changes in tank height and volume, respectively. Instead, one should use alternative quantities dh_E and dv_E, which represent the height and volume increases effectively contributed to the over-all tank. Expressions for dh_E and dv_E are derived herewith. They can be substituted directly for dh and dv as used in Ref. 1. For a cylindrical tank, for which the cone angle $\phi = 0$, the expressions become identical. For small values of ϕ the error is negligible. In fact, for the numerical results presented in Fig. 6 of Ref. 1 the largest error in calculation is less than 0.05%. Thus, the conclusions of Ref. 1 are not changed.

Discussion

Consider a cone frustum, the cross-sectional view of which is represented by *pqrs* in Fig. 1. An element of infinitesmal height h, which in the limit becomes dL, is represented by *abcd*. The shell is of infinitesimal thickness T, so that membrane theory is applicable to its behavior under strain (no stress can be developed perpendicular to the tangent plane at any point on the shell, and no bending moment can be set up). Strain in the element *abcd* produces small deflections as follows:

1) Due to circumferential strain ϵ_θ, the element *abcd* expands and deflects to configuration *efg′h′*, where $g'h' = cd(1 + \epsilon_\theta)$, and $ef = ab(1 + \epsilon_\theta)$, while eh' remains equal to ad.

2) Due to meridional strain ϵ_ϕ, the element *efg′h′* stretches and deflects into the configuration *efgh* where $eh = eh'(1 + \epsilon_\phi)$, while gh remains equal to $g'h'$.

Thus, the volume of the element increases by an actual amount dv to become the volume represented by *efgh*. However, the volume of the frustum *pqrs* increases by an effective amount dv_E to become equal to that represented by *pqbfgmtunheap*. The volume represented by *cdsr* translates as a rigid body into position *mnut* without change. The effective volume increment dv_E is, therefore, equal to the volume of a solid obtained by rotating the shaded area *cbfgmnheadc* about the cone's axis of symmetry. The quantities dv_E and dv are not equal. In Ref. 1 dv and dh were used by mistake for integration between limits to obtain expressions for the total volume and height increases, respectively, of a conical frustum. The correct procedure is to integrate dv_E and dh_E instead.

Using the same nomenclature as Ref. 1—i.e., n_θ and n_ϕ are circumferential and meridional running skin loads, respectively—the expressions for dh_E and dv_E can be written as follows:

$$dh_E = (1/T \cos^2\phi)(n_\phi/E_{Mh} - n_\theta/E_{Hh})dL \qquad (1)$$

$$dv_E = (\pi r^2/T \cos^2\phi)(n_\phi/E_{Mv} + n_\theta/E_{Hv})dL \qquad (2)$$

where $E_{Mh} = E\phi$, $E_{Hh} = E_\theta/\nu_{\phi\theta}$, $E_{Mv} = E_\phi/(1 - 2\nu_{\theta\phi})$, and $E_{Hv} = E_\theta/(2 - \nu_{\phi\theta})$; E_ϕ and E_θ are Young's moduli in the two principal directions, and $\nu_{\phi\theta}$ and $\nu_{\theta\phi}$ are the Poisson coefficients for anisotropic material.

Equations (1) and (2) herewith are equivalent to Eqs. (3) and (4) of Ref. 1 and may be substituted directly in all subsequent parts of Ref. 1.

On page 14 of Ref. 2 Leknhitskii states that $E_\theta\nu_{\theta\phi} = E_\phi\nu_{\phi\theta}$ always. Making use of this theorem, it can be seen that when $\phi = 0$ the expressions for dh and dv as given in Eqs. (3) and (4) of Ref. 1 become identical with those of Eqs. (1) and (2) given herewith for dh_E and dv_E. Thus, for small values of ϕ the discrepancy becomes negligible.

References

[1] Pengelley, C. D., "Natural Frequency of Longitudinal Modes of Liquid Propellant Space Launch Vehicles," *Journal of Spacecraft and Rockets*, Vol. 5, No. 12, Dec. 1968, pp. 1425–1431.

[2] Lekhnitskii, S. G., *Theory of Elasticity of an Anisotropic Elastic Body*, Holden-Day, San Francisco, Calif., 1963.

Erratum: "Orbital Gyrocompassing Heading Reference"

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R. A. CARLSTROM of Huntington, New York was kind enough to point out an error in a figure I prepared. Figure 1 should replace Fig. 4 of the subject paper.

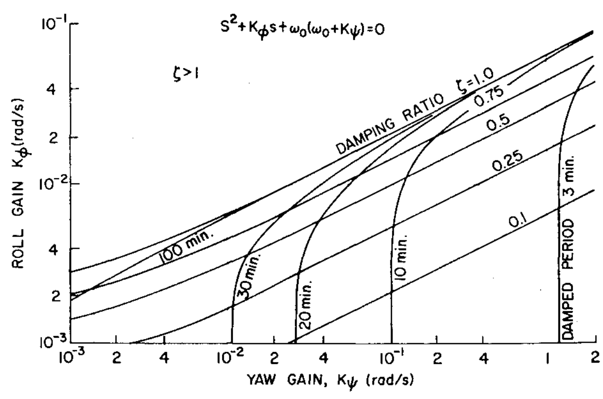

Fig. 1 Natural period and damping of gyro compass as a function of gains in Fig. 3.

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